Decrumpling or TVSD model explains why the universe is accelerating today

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Abstract

Within the framework of a model universe with time variable space dimension (TVSD), known as decrumpling or TVSD model, we show the present value of the deceleration parameter is negative implying that the universe is accelerating today. Our study is based on a flat universe with the equation of state parameter to be $\omega(z=0) \approx -1$ today. More clearly, decrumpling model tells us the universe is accelerating today due to the cosmological constant which is the simplest candidate for the dark energy.

In recent years the exploration of the universe at redshifts of order unity has provided information about the time evolution of the expansion rate of the universe. Observations indicate that the universe is presently undergoing a phase of accelerated expansion [1]. The goal of this letter is to show a model universe with time variable space dimension, known as decrumpling model, explains why the universe is accelerating today. Our approach to conclude this result is to compute the deceleration parameter of the model and to show its present value is negative, implying the universe is accelerating today. For more details about decrumpling or TVSD model see Refs. [2]-[9].

We will use the natural units system that sets k_B, c, and $\hbar$ all equal to one, so that $\ell_P = M_P^{-1} = \sqrt{G}$. To read easily this letter we also use the notation D_t instead of $D(t)$ that means the space dimension D is as a function of time.

In Refs.[2]-[9], the decrumpling or TVSD model has been studied. Assume the universe consists of a fixed number $\mathcal{N}$ of universal cells having a characteristic length δ in each of their dimensions. The volume of the universe at the time t depends on the configuration of the cells. It is easily seen that

$$\mathrm{vol}_{D_t}(\mathrm{cell}) = \mathrm{vol}_{D_0}(\mathrm{cell})\delta^{D_t - D_0}, \tag{1}$$

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where the t subscript in D_t means that D to be as a function of time, i.e. $D(t)$.

Interpreting the radius of the universe, a, as the radius of gyration of a crumpled "universal surface", the volume of space can be written

$$\begin{aligned} a^{D_t} &= \mathcal{N}\,\mathrm{vol}_{D_t}(\mathrm{cell}) \\ &= \mathcal{N}\,\mathrm{vol}_{D_0}(\mathrm{cell})\delta^{D_t-D_0} \\ &= {a_0}^{D_0}\delta^{D_t-D_0} \end{aligned} \tag{2}$$

or

$$\left(\frac{a}{\delta}\right)^{D_t} = \left(\frac{a_0}{\delta}\right)^{D_0} = e^C, \tag{3}$$

where C is a universal positive constant. Its value has a strong influence on the dynamics of spacetime, for example on the dimension of space, say, at the Planck time. Hence, it has physical and cosmological consequences and may be determined by observation. The zero subscript in any quantity, e.g. in a_0 and D_0, denotes its present value. We coin the above relation as a"dimensional constraint" which relates the "scale factor" of decrumpling model to the spatial dimension. We consider the comoving length of the Hubble radius at present time to be equal to one. So the interpretation of the scale factor as a physical length is valid. The dimensional constraint can be written in this form

$$\frac{1}{D_t} = \frac{1}{C}\ln\left(\frac{a}{a_0}\right) + \frac{1}{D_0}. \tag{4}$$

It is seen that by the expansion of the universe, the space dimension decreases. Time derivative of (3) or (4) leads to

$$\dot{D}_t = -\frac{D_t^2\dot{a}}{Ca}. \tag{5}$$

It can be easily shown that the case of constant space dimension corresponds to when C tends to infinity. In other words, C depends on the number of fundamental cells. For $C \to +\infty$, the number of cells tends to infinity and $\delta \to 0$. In this limit, the dependence between the space dimensions and the radius of the universe is removed, and consequently we have a constant space dimension.

Table 1: Values of C and δ for some values of D_P. Time variation of space dimension today has been also calculated in terms of sec^{-1} and yr^{-1}.

D_P	C	δ (cm)	$\dot{D}\vert_0$ (sec^{-1})	$\dot{D}\vert_0$ (yr^{-1})
3	$+\infty$	0	0	0
4	1678.797	8.6158×10^{-216}	$-1.7374 \times 10^{-20} h_0$	$-5.4827 \times 10^{-13} h_0$
10	599.571	1.4771×10^{-59}	$-4.8648 \times 10^{-20} h_0$	$-1.5352 \times 10^{-12} h_0$
25	476.931	8.3810×10^{-42}	$-6.1158 \times 10^{-20} h_0$	$-1.9299 \times 10^{-12} h_0$
$+\infty$	419.699	ℓ_P	$-6.9498 \times 10^{-20} h_0$	$-2.1931 \times 10^{-12} h_0$

We define D_P as the space dimension of the universe when the scale factor is equal to the Planck length ℓ_P. Taking $D_0 = 3$ and the scale of the universe today to be the present value of the Hubble radius H_0^{-1} and the space dimension at the Planck length to be $4, 10$, or 25, from Kaluza-Klein and superstring theory, we can obtain from (3) and (4) the corresponding value of C and δ

$$\frac{1}{D_P} = \frac{1}{C} \ln\left(\frac{\ell_P}{a_0}\right) + \frac{1}{D_0} = \frac{1}{C} \ln\left(\frac{\ell_P}{{H_0}^{-1}}\right) + \frac{1}{3}, \tag{6}$$

$$\delta = a_0 e^{-C/D_0} = H_0^{-1} e^{-C/3}. \tag{7}$$

In Table 1, values of C, δ and also $\dot{D}_t|_0$ for some interesting values of D_P are given. These values are calculated by assuming $D_0 = 3$ and $H_0^{-1} = 3000 h_0^{-1}\mathrm{Mpc} = 9.2503 \times 10^{27} h_0^{-1}\mathrm{cm}$, where we take $h_0 = 1$.

Let us define the action of the model for the special Friedmann-Robertson-Walker (FRW) metric in an arbitrary fixed space dimension D, and then try to generalize it to variable dimension. Now, take the metric in constant $D+1$ dimensions in the following form

$$ds^2 = -N^2(t)dt^2 + a^2(t) d\Sigma_k^2, \tag{8}$$

where $N(t)$ denotes the lapse function and $d\Sigma_k^2$ is the line element for a D-manifold of constant curvature $k = +1, 0, -1$. The Ricci scalar is given by

$$R = \frac{D}{N^2}\left\{\frac{2\ddot{a}}{a} + (D-1)\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{N^2 k}{a^2}\right] - \frac{2\dot{a}\dot{N}}{aN}\right\}. \tag{9}$$

Substituting from Eq.(9) in the Einstein-Hilbert action for pure gravity in constant $D+1$ dimensions

$$S_G = \frac{1}{2\kappa}\int d^{(1+D)}x\sqrt{-g}R, \tag{10}$$

and using the Hawking-Ellis action of a perfect fluid for the model, the following Lagrangian has been obtained [9]

$$L_I := -\frac{V_{D_t}}{2\kappa N}\left(\frac{a}{a_0}\right)^{D_t} D_t(D_t-1)\left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{N^2 k}{a^2}\right] - \rho N V_{D_t}\left(\frac{a}{a_0}\right)^{D_t}, \tag{11}$$

where $\kappa = 8\pi {M_P}^{-2} = 8\pi G$, ρ the energy density, and V_{D_t} the volume of the space-like sections

$$V_{D_t} = \frac{2\pi^{(D_t+1)/2}}{\Gamma[(D_t+1)/2]}, \quad \text{closed universe, } k=+1, \tag{12}$$

$$V_{D_t} = \frac{\pi^{(D_t/2)}}{\Gamma(D_t/2+1)}{\chi_c}^{D_t}, \quad \text{flat universe, } k=0, \tag{13}$$

$$V_{D_t} = \frac{2\pi^{(D_t/2)}}{\Gamma(D_t/2)}f(\chi_c), \quad \text{open universe, } k=-1, \tag{14}$$

where χ_C is a cut-off and $f(\chi_c)$ is a function thereof (for more details see Ref. [9]). In the limit of constant space dimensions, or $D_t = D_0$, L_I approaches to the Einstein-Hilbert Lagrangian which is

$$L_I^0 := -\frac{V_{D_0}}{2\kappa_0 N}\left(\frac{a}{a_0}\right)^{D_0} D_0(D_0-1)\left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{N^2 k}{a^2}\right] - \rho N V_{D_0}\left(\frac{a}{a_0}\right)^{D_0}, \tag{15}$$

where $\kappa_0 = 8\pi G_0$ and the zero subscript in G_0 denotes its present value. So, Lagrangian L_I cannot abandon Einstein's gravity. Varying the Lagrangian L_I with respect to N and a, we find the following equations of motion in the gauge $N=1$, respectively

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{2\kappa\rho}{D_t(D_t-1)}, \tag{16}$$

$$(D_t-1)\left\{\frac{\ddot{a}}{a} + \left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\left(-\frac{D_t^2}{2C}\frac{d\ln V_{D_t}}{dD_t} - 1 - \frac{D_t(2D_t-1)}{2C(D_t-1)} + \frac{D_t^2}{2D_0}\right)\right\}$$
$$+\kappa p\left(-\frac{d\ln V_{D_t}}{dD_t}\frac{D_t}{C} - \frac{D_t}{C}\ln\frac{a}{a_0} + 1\right) = 0. \tag{17}$$

Using (16) and (17), the evolution equation of the space dimension can be obtained by

$$\dot{D_t}^2 = \frac{D_t^4}{C^2}\left[\frac{2\kappa\rho}{D_t(D_t-1)} - k\delta^{-2}e^{-2C/D_t}\right]. \tag{18}$$

The continuity equation of the model universe with variable space dimension can be obtained by (16) and (17)

$$\frac{d}{dt}\left[\rho\left(\frac{a}{a_0}\right)^{D_t} V_{D_t}\right] + p\frac{d}{dt}\left[\left(\frac{a}{a_0}\right)^{D_t} V_{D_t}\right] = 0. \tag{19}$$

All above equations, from (1) to (19), are written and discussed in Refs.[2]-[9]. Let us here calculate the present value of the deceleration parameter q_0 in this model. We use (17) in the case of flat universe, i.e. $k = 0$. Therefore, we get

$$\begin{aligned} q_0 &\equiv \left.\frac{-\ddot{a}\,a}{\dot{a}^2}\right|_0 \\ &= -\frac{D_0^2}{2C}\left.\frac{d\ln V_D}{dD}\right|_{D=D_0} - 1 - \frac{D_0(2D_0-1)}{2C(D_0-1)} \\ &+ \frac{D_0}{2} - \frac{p_0 D_0^2}{2\rho_0}\left(\frac{1}{C}\left.\frac{d\ln V_D}{dD}\right|_{D=D_0} - \frac{1}{D_0}\right), \end{aligned} \tag{20}$$

where p_0 and ρ_0 are the present value of the pressure and the energy density respectively, satisfying the present equation of state in the model

$$p_0 = \omega_0 \rho_0. \tag{21}$$

Here ω_0 is the parameter of the equation of state at the present time. Therefore we can rewrite (20) in terms of ω_0 as

$$q_0 = \frac{D_0^2}{2}(1+\omega_0)\left(-\frac{1}{C}\left.\frac{d\ln V_D}{dD}\right|_{D=D_0} + \frac{1}{D_0}\right) - 1 - \frac{D_0(2D_0-1)}{2C(D_0-1)}. \tag{22}$$

This equation can be directly obtained by the time derivative of (16) in the case of a flat universe.

Concerning the dynamics in a flat universe, the authors of Ref.[10] conclude on the validity of the Cosmic Concordance version of the ΛCDM Model that is $\Omega_M \approx 0.3$, $\Omega_\Lambda \approx 0.7$, $\omega(z=0) \approx -1$ and no rapid evolution of the

equation of state parameter. In the limit of constant space dimension, i.e. $C \to +\infty$, from Eq.(22) the deceleration parameter today is given by

$$q_0 = \frac{D_0}{2}(1+\omega_0) - 1. \tag{23}$$

Taking $\omega_0 \approx -1$ and $D_0 = 3$, the value of q_0 in the universe with constant space dimension satisfies $q_0 \approx -1$. In the case of decrumpling model, by considering $\omega_0 \approx -1$ and $D_0 = 3$, the value of the deceleration parameter satisfies

$$q_0 \approx -1 - \frac{15}{4C}, \tag{24}$$

(see Eq.(22) and the values of C given in Table 1). Therefore, decrumpling model explains the universe is accelerating today, i.e. $q_0 < 0$ or $\ddot{a} > 0$ and in this model the reason for the present cosmic acceleration of the universe is due to the cosmological constant which is the simplest candidate of the dark energy. In the case of matter-dominated universe, i.e. $p = 0$, we do not analyze Eq.(22) because supernovae searches have shown that a simplest matter-dominated and decelerating universe should be ruled out.

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